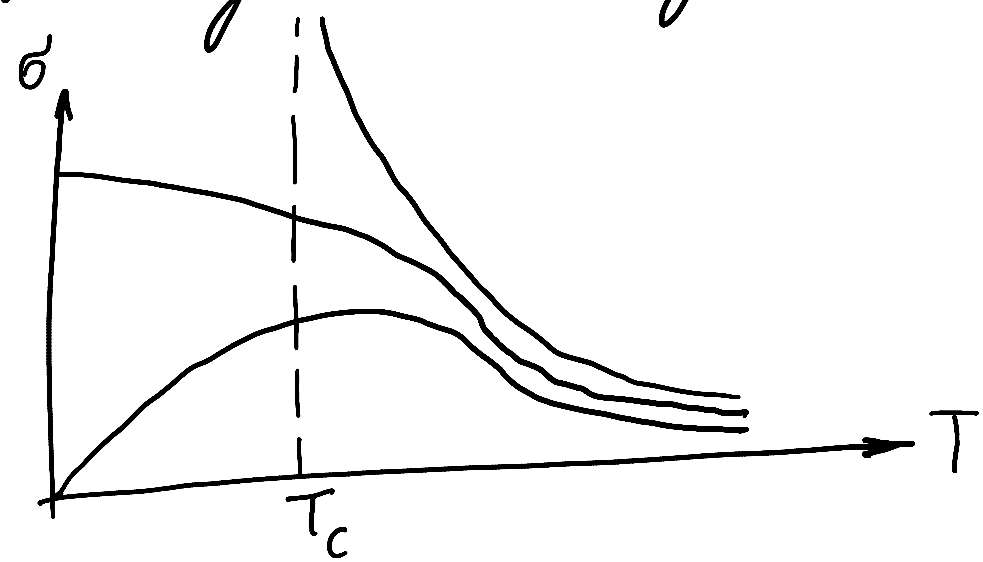
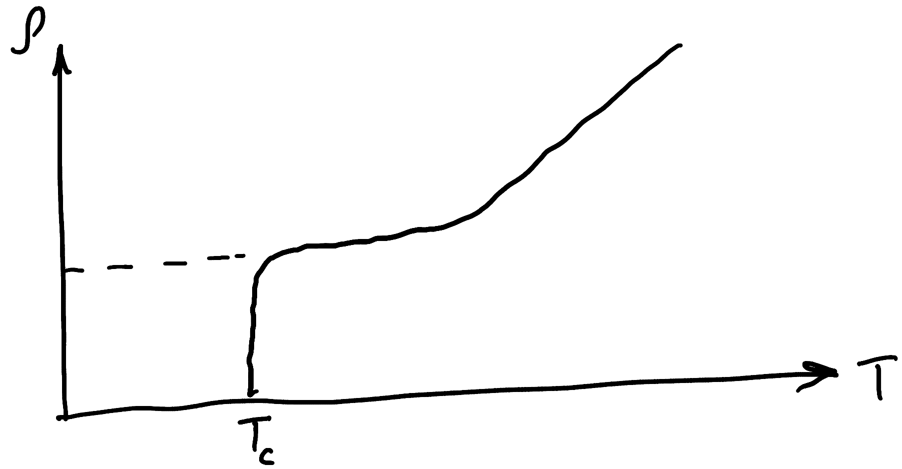
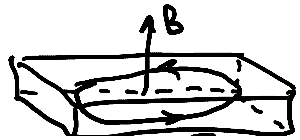
Superconductivity. Phenomenology and thermodynamics

Vanishing of resistivity





*) Meissner effect
(= Meissner-Ochsenfeld effect)
– the vanishing of magnetic field inside of a superconductor
(in fact, applies to <u>type I</u> superconductors)



Note: this is not a consequence of vanishing resistivity

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$$\mathcal{E} = \frac{1}{c}\frac{\partial \varphi}{\partial t}$$

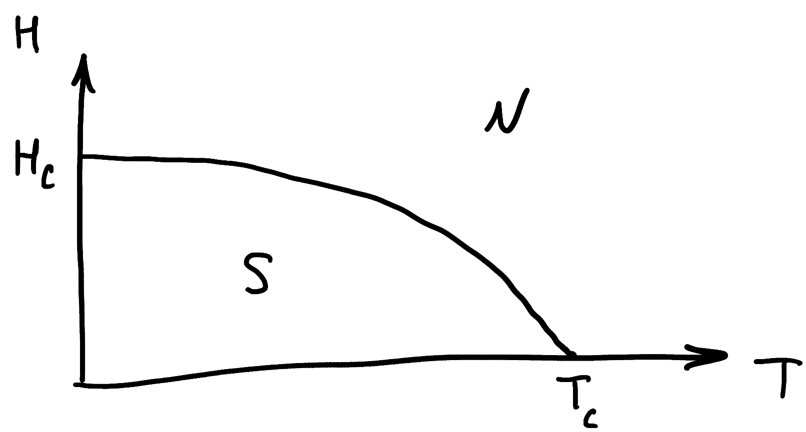
voltage induced in a closed loop = electromotive force

One needs $\mathcal{E} = 0$.
Otherwise, $I = \frac{\mathcal{E}}{R} \to \infty$
Thus, the system must induce current to screen the change of the external magnetic field

Thus $\varphi = \text{const}$, so long as $\sigma = \infty$

However, in a superconductor, unlike an ideal metal, $\varphi = 0$ – a thermodynamic state

*) Supercondunctivity is destroyed by a sufficiently strong magnetic field



$$H_c(T) = H_c\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Superconductors

Type I

Type II

Type I | Type II

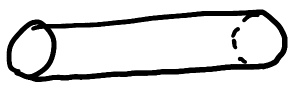
(All element SC (Al, Cd, Ga, ...) except Nb)

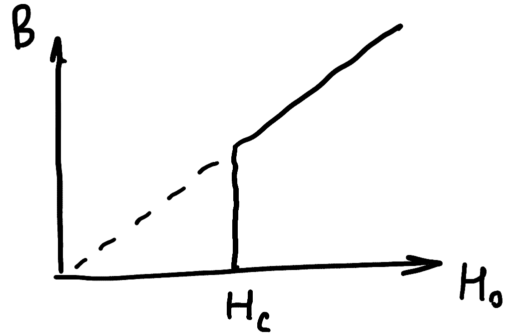
Meissner effect

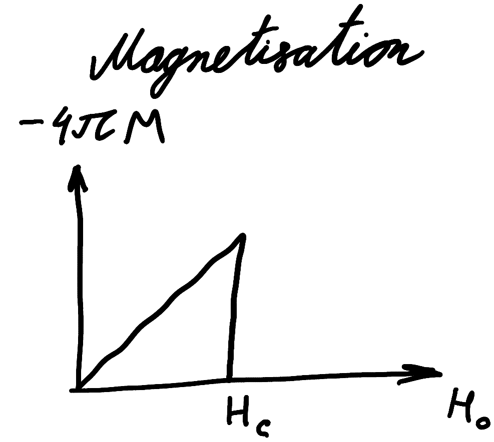
Alloys

Magnetic field partially penetrates for $H_{c1} < H < H_{c2}$

Consider Type I superconductors





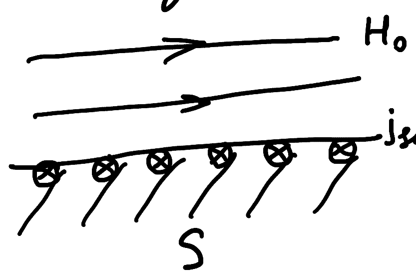


$$\vec{B} = \vec{H}_0 + 4\pi \vec{M}$$

Inside a superconductor there is no magnetic field, and on it's surface there are no magnetic charges ($\operatorname{div} \vec{B} = 0$)

→ The normal magnetic field vanishes

→ The magnetic field is always tangential

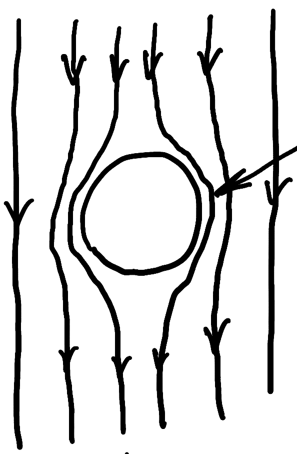


$$j_{surf} = \frac{c}{4\pi} H_0$$

$$\vec{j}_{surf} = \frac{c}{4\pi} \vec{n} \times \vec{H}_0$$

P... tration of magnetic field into a

Penetration of magnetic field into a superconductor



H_0

Magnetic field here is stronger and will reach H_c before H_0 reaches H_c

The ball may be in a mixed state

$$H_c \to H_m = \frac{H_0}{1-n} \qquad (n < 1)$$

n - demagnetising factor

Thermodynamics

In type I superconductors $\vec{M} = -\frac{\vec{H}}{4\pi}$

When $\vec{H}$ changes, $\vec{H} \to \vec{H} + d\vec{H}$, it does the amount of work

$$-\vec{M}\, d\vec{H} = \frac{1}{4\pi} \vec{H}\, d\vec{H}$$

If $F(H)$ is the free energy of the superconductor,

$$F(H) - F(0) = \frac{H^2}{8\pi}$$

If it is favourable to switch to the normal state, the superconductor will

Thus $F = F + \frac{H_c^2}{}$ (*)

Thus, $$\boxed{F_n = F_s + \frac{H_c^2}{8\pi}} \qquad (*)$$

Find heat capacitance from the free energy

$$F = U - TS$$

$$dF = dU - TdS - SdT = -\underset{\text{work}}{dW} - SdT$$

$$\rightarrow S = -\left(\frac{\partial F}{\partial T}\right)_W$$

Use formula $(*)$ above

$$S_s - S_n = \frac{H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)_W$$

Several consequences:

o) Because $S = 0$ for $T = 0$ (Nernst theorem)

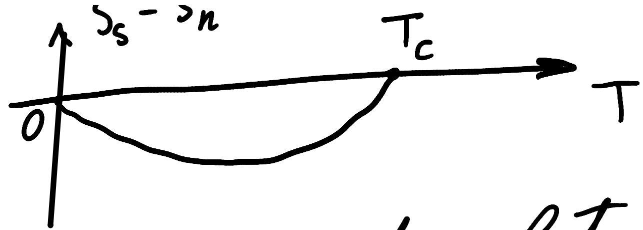
$$\left(\frac{\partial H_c}{\partial T}\right)_{T=0} = 0$$

o) $\frac{\partial H_c}{\partial T} < 0$ – experimental fact

$$\rightarrow S_s < S_n$$

→ Superconductivity is a more ordered state than normal metal

o) For $T = T_c$ $\quad H_c = 0$ and $S_s = S_n$



→ The transition between S and N occurs without absorbing or emitting heat (at T_c)

On the other hang, if $T < T_c$ and the system goes from N to S due to changing the magnetic field, then it releases heat.

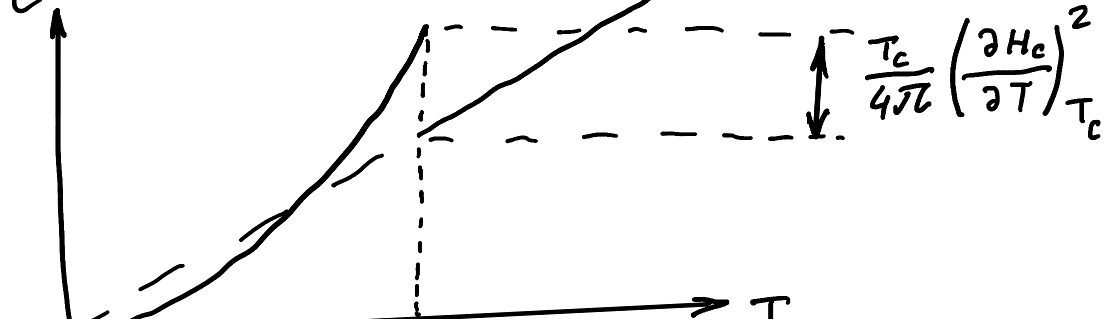
Heat capacity

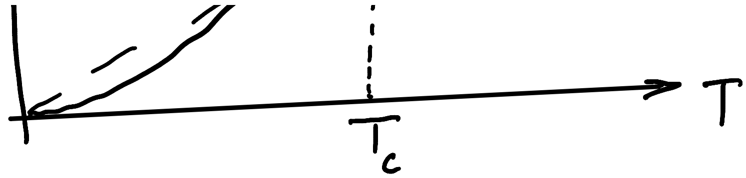
$$C = T \frac{\partial S}{\partial T}$$

$$C_s - C_n = \frac{T}{4\pi}\left[\left(\frac{\partial H_c}{\partial T}\right)^2 + H_c \frac{\partial^2 H_c}{\partial T^2}\right]$$

$$\boxed{C_s - C_n = \frac{T_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)^2_{T_c}}$$ – Rutgers formula

– jump of the heat capacity at the S-N transition





London equations

$n = n_s + n_n$

„Superconductive" electrons

„Normal" electrons

(Electrons „ in the condensate")

$$n_s m \frac{d\vec{v}_s}{dt} = n_s e \vec{E}$$

1-st London equation

$$\boxed{\vec{E} = \frac{d}{dt}\left(\Lambda \vec{j}_s\right)}\,, \text{ where } \Lambda = \frac{m}{n_s e^2}$$

and $\vec{j}_s = n_s e \vec{v}_s$

The kinetic energy of the „super" electrons:

$$\left.\begin{aligned} W_{kin} &= \frac{n_s m v_s^2}{2} = \frac{m j_s^2}{2 n_s e^2} \\ \text{Maxwell equation} \quad \operatorname{rot} \vec{H} &= \frac{4\pi}{c} \vec{j}_s \end{aligned}\right\} \rightarrow$$

$$\rightarrow \quad W_{kin} = \frac{\lambda^2}{8\pi} (\operatorname{rot} \vec{H})^2$$

where $\lambda^2 = \frac{m c^2}{4\pi n_s e^2}$

The density of the magnetic and kinetic energies:

$$F_{sH} = F_{s0} + \frac{1}{8\pi} \int \left[H^2 + \lambda^2 (\operatorname{rot} \vec{H})^2 \right] dV$$

$$F_{sH} = F_{s0} + \frac{1}{8\pi}\int \Big[H^2 + \ldots \Big]$$

Consider a variation of this free energy

Variation = at each point the magnetic field is modified : $\vec{H} \to \vec{H} + \delta\vec{H}$. at the boundary $\delta\vec{H} = 0$

$$\delta F_s = \frac{1}{8\pi}\int \left(2\vec{H}\,\delta\vec{H} + 2\lambda^2 \,\mathrm{rot}\,\vec{H}\;\mathrm{rot}\,\delta\vec{H}\right) dV$$

Use that $\underset{\mathrm{rot}\,\hat{H}}{\vec{a}}\;\mathrm{rot}\,\underset{\delta\vec{H}}{\vec{b}} = \vec{b}\;\mathrm{rot}\,\vec{a} - \mathrm{div}\,\vec{a}\times\vec{b}$

$$\delta F_s = \frac{1}{4\pi}\int_V \left[\vec{H} + \lambda^2\,\mathrm{rot}\,\mathrm{rot}\,\vec{H}\right]\delta\vec{H}$$

$$- \frac{1}{4\pi}\underbrace{\int_V \mathrm{div}\left(\mathrm{rot}\,\vec{H}\times\delta\vec{H}\right)}_{=\int_{\text{Surface}} \mathrm{rot}\,\vec{H}\times\delta\vec{H}\;d\vec{S} = 0} = 0$$

Because the field on the surface is fixed when finding the variational derivative

Then $\boxed{\vec{H} + \lambda^2\,\mathrm{rot}\,\mathrm{rot}\,\vec{H} = 0}$ – 2nd London equation

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If we use $\mathrm{rot}\,\vec{H} = \frac{4\pi}{c}\vec{j}$

Together with the 2nd London equation this gives

$$\mathrm{rot}\,\vec{A} + \frac{4\pi\lambda^2}{c}\,\mathrm{rot}\,\vec{j} = 0$$

$$\vec{j} = -\frac{c}{4\pi\lambda^2}\vec{A} \qquad (*)$$

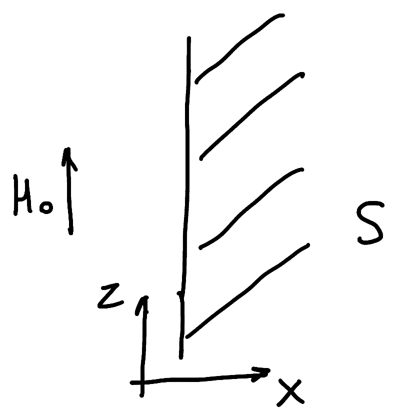
if we choose gauge $\mathrm{div}\,\vec{A} = 0$, $\vec{A}\vec{n} = 0$

if we choose gauge div $\vec{A} = 0$,

$$\Lambda = \frac{4\pi\lambda^2}{c^2}$$

(*) follows also from the first London equation

Penetration depth



$$\vec{H} + \lambda^2 \operatorname{rot}\operatorname{rot}\vec{H} = 0$$

$$H + \lambda^2 \frac{d^2 H}{dx^2} = 0$$

$$H = H_0 e^{-\frac{x}{\lambda}}$$

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{\frac{1}{2}}$$ – penetration depth

$$j_s = \frac{c H_0}{4\pi\lambda} e^{-\frac{x}{\lambda}}$$

Empiric formula:

$$\lambda(T) = \frac{\lambda(0)}{\left(1 - \left(\frac{T}{T_c}\right)^4\right)^{\frac{1}{2}}}$$